Nonlinear wave dynamics in the presence of mud-induced dissipation on Atchafalaya Shelf, Louisiana, USA

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ABSTRACT

The interplay between wave nonlinearity and mud-induced dissipation is studied here using wave and sediment transport measurements collected in winter 2008, on the Atchafalaya Shelf, Louisiana, between the 8-m and the 4-m isobaths. This study focuses on the relatively energetic storm that occurred on March 4th (2-m wave height in 8-m water depth), which caused significant bed reworking and left in its wake a 20-cm layer of hindered-settling fluid mud. While the net wave dissipation rate was maximal during the hindered-settling phase after the storm, consistent with previous observations, significant dissipation was observed throughout the storm duration, with a secondary maximum associated with the peak of the storm and the maximum bed-reworking effects. The effects of mud-induced dissipation on the nonlinear shoaling process are investigated here using TRIADS, a newly-developed spectral model for nonlinear shoaling of waves. With mud-parameter values estimated using a crude inverse modeling approach, the model was used to separate the effects of mud-induced dissipation from nonlinear transfers, during both the erosion and deposition phases of bed reworking. Numerical simulations show that nonlinear transfers are always active, even under strong dissipation regimes. Remarkably, they also suggest that heavy mud-induced dissipation causes the nonlinear energy cascade to revert direction, with bulk transfers occurring from high-frequency to low-frequency spectral bands. This supports the hypothesis that the interplay between mud-induced dissipation, which is much more significant than the bottom dissipation over sandy beds, and wave nonlinearity can drain energy from the entire spectrum, and not just from the frequency bands that interact directly with the seafloor. The decay of high-frequency variance induced by the reversal of spectral flux in effect reduces the nonlinearity of the wave field.

1. Introduction

The interplay between mud-induced wave energy dissipation (e.g., Gade, 1958; Wells and Coleman, 1981; Sheremet et al., 2005a; Kaihatu et al., 2007; Elgar and Raubenheimer, 2008; Safak et al., 2013a) and nonlinear wave propagation in shallow water is not fully understood. It has been hypothesized that, due to nonlinear interactions, mud-induced dissipation affects not only long waves that interact directly with the bottom, but also high-frequency waves, in effect draining the energy from the entire spectrum (Sheremet and Stone, 2003). Such a generalized interaction could affect wave propagation in unexpected ways, for example, by suppressing the development of steep wave fronts and associated wave breaking, or generating diffraction through non-uniform mud distribution (Kaihatu et al., 2007). Mud-induced dissipation is the response of waves to a muddy bed, whether through the formation of wave-supported fluid mud layers in the waning phase of a storm, or through phase changes of the muddy bed state such as liquefaction during the storm. Wave-supported fluid mud layers are routinely observed when high-concentration fluid mud layers are formed due to hindered settling of initially suspended cohesive sediment (Traykovski et al., 2000; Wright et al., 2001; Sheremet et al., 2011a; Safak et al., 2013a), eventually decaying to laminar flows with high viscosity and small velocities. The processes associated with bed reworking at the peak of a storm are less studied. The term “bed reworking” is used here as a generic name subsuming processes such as expansion, liquefaction, and fluidization (see discussion in Sahin et al., 2012). It excludes the formation of fluid mud layers through hindered settling, a process that typically occurs during the waning phase of a storm.
Although the problem of the interplay between nonlinear wave-wave interactions and band-limited mud-induced dissipation has been considered before (Sheremet et al., 2005b, 2011a; Alam et al., 2011), its details are yet to be resolved, in part because of incompleteness of data, but also in part due to nonlinear model limitations such as unidirectional representation (Sheremet et al., 2011a; Safak et al., 2013a). Here, we revisit the problem using new data and new modeling technology.

Muddy-bed reworking by waves has been the focus of several large-scale field experiments conducted in 2008 and 2010 on the Atchafalaya Shelf and the Louisiana Chenier Plain. These experiments produced comprehensive data sets discussed since in a number of papers (e.g., Elgar and Raubenheimer, 2008; Safak et al., 2010, 2013b; Sahin et al., 2013; Traykovski et al., 2015; Safak, 2016). The data set used here contains detailed information about directional characteristics of surface waves, as well as muddy bed transformation. In particular, the data set used here contains information not only about the dissipation effects of the waning-stage fluid mud layers, but also about bed reworking effects during the peak of the storm. Therefore, wave-mud interaction throughout the entire storm duration is examined. The main goal of the paper is to investigate wave dissipation induced by mud-reworking stages of a storm other than the fluid-mud stage, which occurs at the end of the storm. Such an investigation requires a numerical tool to separate the contribution of nonlinearity and dissipation, as their effects on wave propagation have similar order of magnitude (e.g., Sheremet et al., 2011a) and can be separated only locally (as rates of change, derivatives) and not globally (as global contributions).

Accurate modeling of wave transformation on the Atchafalaya Shelf is difficult, due to the large-scale, shallow-water environment (the 10-m isobath can be as far as 50 km offshore). Such efforts are few (e.g., Winterwerp et al., 2007; Rogers and Holland, 2009) and typically use directional phase-averaged models and thus are not adequate for the Atchafalaya shallow shelf. Alternatively, attempts to account for the triad interactions (Sheremet et al., 2005b, 2011a; Elgar and Raubenheimer, 2008; Safak et al., 2013a), which dominate nonlinear shallow water wave transformation, have been hampered by the use of unidirectional models. Unidirectional models previously showed skill in reproducing observed mud-induced dissipation (Sheremet et al., 2011a; Safak et al., 2013a), a feature useful, for example, for the development of fast numerical procedures for estimating mud characteristics. However, the natural wave-shoaling process is fundamentally directional, and although directional numerical simulations bring additional complexity and numerical effort, they provide a more realistic framework for the description of nonlinear effects. In a recent application of a phase-resolving model based on idealized directionality with zero and oblique incidence angles, a zero initial angle in the model never returned the best comparison with the data which showed that wave directionality may help to capture some of the missing features of wave propagation over muddy shelves (Liao et al., 2015). A secondary goal of the paper is to take a further step in the complexity of the numerical representation, by using a directional triad interaction model. Recently, a phase-resolving, directional triad interaction model was developed by Davis et al. (2014) and Sheremet et al. (2016), which has the potential of eliminating the unidirectional wave constraint. The development of the TRIADS model engine provides the opportunity to examine the problem under the assumptions of alongshore uniform bathymetry, but with broad directional spectra.

Brief descriptions of the experiment setup and instrumentation, and of the numerical model used here are given in Sections 2 and 3, respectively. Sections 4 and 5 present the analysis of the observations and numerical simulations. Section 6 summarizes our findings.

2. Experiment site and instrumentation

The field observations discussed here were collected on the muddy Atchafalaya Shelf at the northern Gulf of Mexico (Fig. 1) between February 23rd and March 7th, 2008. This time interval typically coincides with high discharge of the Atchafalaya River and energetic waves on the Atchafalaya Shelf, both processes driven by frequent cold atmospheric fronts passing over the region (Walker and Hammack, 2000). The Atchafalaya Shelf is a large-scale, shallow basin with a slope of

![Map of the Atchafalaya Shelf](image-url)
The linear cross-shore energy flux is defined rigorously in Section 3. It is used here as the quantity of choice for the analysis of dissipation because it incorporates the effects of shoaling: in other words, in the absence of dissipation and nonlinear transfers, both modal fluxes and the total flux \( F(x) \) are constant. The rate \( D \) represents net dissipation rate, i.e., the net result of all the effects that change \( F \), combined. For the problem discussed here, the \( F \)-altering effects considered are mud-induced dissipation and nonlinear transfers. It can be shown in the convention of Equation (1), \( D < 0 \) represents net dissipation, and \( D > 0 \) represents net growth. Definition 1 is used to estimate the net total flux dissipation between sites P1 and P3, by approximating the transect as cross-shore and setting \( \Delta x = 8 \) km, the distance between the sites, and using the total energy fluxes estimated at sites P1 and P3.

3. Numerical model

The current implementation of TRIADS (Davis et al., 2014) is based on the nonlinear mild slope formulation of Agnon and Sheremet (1997). The formulation assumes a cylindrical (laterally uniform) beach with a mild slope in the cross-shore direction. Let \( x \) be the cross-shore, and \( y \) the alongshore direction, with the vertical coordinate \( z \) and the local depth \( h(x) \). The free surface elevation \( \eta(x,y,t) \) is described as

\[
\eta(x,y,t) = \sum_j a_j(x,t) \exp[i(\theta_j(x,t) + k_jy - \omega_jt)],
\]

where the index (mode)

\[
J = (\omega_j, k_j)
\]

represents a discretization of the two-dimensional Fourier space \( (\omega, k) \in \mathbb{R}^2 \); and \( a_j \) and \( \theta_j(x,t) \) are the modal amplitude and cross-shore phase, with \( a_j = |a_j| \) and \( -J = (-\omega, -k) \) (the asterisk denotes the complex conjugate). Under the assumption of the cylindrical beach, the frequency \( \omega \) and the alongshore wave number \( k \) are invariants of propagation. At a given depth \( h(x) \), the frequency and wave numbers satisfy the equations

\[
a^2 = gK \tan hK; \quad K^2 = k^2 + \kappa^2; \quad k = \frac{dh}{dx}
\]

with \( k \) the cross-shore component of the wave number vector \( \mathbf{K} = (k, \kappa) \). The local direction of propagation of mode \( J \) is uniquely determined by relations 4. In Equations (2)–(4), the functions \( h, K, \kappa \), and \( a \) are assumed to vary slowly with \( x \) and \( t \) (the mild-slope assumption). The directional evolution equation for the amplitudes of the Fourier modes is (e.g., Agnon and Sheremet, 1997; Davis et al., 2014):

\[
\frac{dX}{dx} = \frac{1}{2} \sum_j b_j \frac{1}{2} \sum_{P,Q} W_{j,P,Q} a_{P,Q} e^{-i \Delta_k \cdot \mathbf{r}} \delta_{Q,j-P} + 2i \sum_{P,Q} W_{j,P,Q} a_{P,Q} e^{-i \Delta_k \cdot \mathbf{r}} \delta_{Q,j-P} + \Delta_k \cdot \mathbf{r} \theta_q - \theta_Q
\]

where

\[
b_j = a_j C_j^2; \quad C_j = \frac{\partial P_j}{\partial P_j} \left( \frac{h}{K} \right); \quad \Delta_k \cdot \mathbf{r} = \theta_\Delta \frac{\Delta \theta}{\Delta \phi} - \theta \phi,
\]

with \( C_j \) the cross-shore component of the modal group velocity \( C_j h_P = b_j \) is the complex conjugate of \( b \). In Equation (5) the summation in the right-hand side is carried over all the modes in the discretization of the half-plane \( (\omega, k) \), with \( \omega > 0 \). The modal change rate \( c_j \) can be used to represent any process of dissipation \( (c_j < 0) \) or growth \( (c_j > 0) \) that affects directly mode \( J \). The triad selection criterion is represented by the Kromekker symbol

| Table 1: Locations of the instrumented platforms (Fig. 1). |
|---------------------|---------------------|
| Site    | Latitude (North deg) | Longitude (West deg) |
| P1      | 29.1969              | 91.6122              |
| P2      | 29.2240              | 91.5801              |
| P3      | 29.2596              | 91.5711              |

This table lists the locations of the instrumented platforms. The distances between the sites are \( 8 \) km, and the spectral peak (swell band) is typically the result of nonlinear interactions which transfer energy toward higher frequencies; the infraband is strongly influenced by the so-called nonlinear ‘sum’ interactions which transfer energy toward higher frequencies; the infraband is typically the result of nonlinear ‘difference’ interactions. For the storm discussed here, \( f_{peak} = 0.13 \) Hz. The Ursell number, defined here as \( U_r = \frac{f_{peak}}{f_{shear}} \), was used as a measure of nonlinearity of the wave field, where \( a_1 \) is the significant wave amplitude, \( k_{peak} \) is the wave number of the spectral peak, and \( h \) is the local depth. \( U_r \approx 1 \) for linear waves and \( U_r \approx 1 \) for nonlinear waves (for example, \( U_r \approx 1 \) was found for a wave in advanced deformation immediately preceding plunging breaking; Sheremet et al., 2011b). Note, however, that the Ursell number is only a qualitative indicator of the nonlinear character of a wave field, defined this way for mathematical convenience. A realistic measure of the nonlinearity should involve higher-order wave statistics such as model phase correlations, or equivalently, wave skewness and asymmetry (see below).

The rate of net change in cross-shore energy flux \( F(x) \) (whether modal or total flux), where \( x \) is the cross-shore position, is

\[
D = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \frac{F(x + \Delta x) - F(x)}{\Delta x}.
\]
\[ \delta_{J,Q} = \begin{cases} 1 & \text{if } J \neq P = Q, \\ 0 & \text{otherwise.} \end{cases} \] (7)

The linear part of Equation (5) represents the conservation of the linear cross-shore energy flux (note that \(|b_j|^2 = c_j|a_j|^2\)). The interaction coefficient is

\[ W_{J,P,Q} = \sqrt{\frac{\sigma_P}{\sigma_P \sigma_Q}} (c_j c_P c_Q)^{-1/2} \left( \pm 2K_P K_Q + K_P^{\sigma_Q} K_Q^{\sigma_P} + K_P^{\sigma_P} K_Q^{\sigma_Q} \right) \right)^{1/2} \]

with \( K_P K_Q = k_P k_Q + \omega_P \omega_Q = K_P^{\sigma_P} K_Q^{\sigma_Q} \), where \( \omega_P \omega_Q = \text{the angle between the wave numbers of modes } P \text{ and } Q \).

Equation (5) can be transformed into an energy equation through straightforward manipulations. Multiplying the equation by \( b_j \) and denoting by \( F_j \) the linear, cross-shore energy flux for mode \( J \) (Equation (6))

\[ F_j = \frac{1}{2} |b_j|^2 = \frac{1}{2} |a_j|^2 c_j \] (10)

obtains

\[ \frac{dF_j}{dx} = \epsilon_j F_j + \text{nonlinear terms.} \] (11)

Equation (11) illustrates a couple of important points: 1) if there is no direct dissipation/growth mechanism (\( \epsilon_j = 0 \)), and if the nonlinear terms are negligible, the modal energy flux \( F_j \) and the total flux \( F = \sum_j F_j \) are conserved during propagation. Therefore, the only effects that can change the modal fluxes are dissipation/growth, and nonlinearity. Remarkably, it can be shown that nonlinear interactions alone do not modify the total flux. 2) various direct dissipation/growth mechanisms can combine in quite complicated ways through the propagation history. These effects are non-local, in the sense that the state of the wave-field at a given position depends on the history of the evolution and the forcing. Because the dissipation is assumed to act as a linear mechanism, various mechanisms can be readily accommodated. In this study, we investigate the effect of two mechanisms

\[ \epsilon_j = \epsilon_j^l + \epsilon_j^r, \] (12)
where $\epsilon_J^b$ represents the contribution of depth-limited wave breaking, and $\epsilon_J^m$ represents mud-induced dissipation. Breaking is parameterized following Kaihatu and Kirby (1995), with parameter values $B = 1$, $\gamma = 0.7$ and $F = 0.5$ (see also Sheremet et al., 2011b, 2016). Mud-induced dissipation is implemented as detailed in (Kaihatu et al., 2007; Sheremet et al., 2011a), using the model first proposed by Ng (2000).

The focus of this study is the interplay between mud-induced dissipation and nonlinear wave-wave interactions. A detailed analysis of the sensitivity of the model to the values of characteristic mud parameters (not presented here) essentially reproduced the results of the analysis conducted by Tahvildari and Kaihatu (2011). Although the model is sensitive to the setting of these free parameters, it is not so sensitive that inversion is not possible in most cases. It is important to note that in our simulations, within a physically meaningful range of mud parameter values, the effect of nonlinear interactions was always present. Therefore, despite the sensitivity of the model to dissipation rates and related parameters, the basic features of wave nonlinearity and its interaction with dissipation were still preserved.

The relation between the total, net change rate $D$ defined in Equation (1) and the modal dissipation $\epsilon$ can be retrieved from Equation (11) by summing over all the modes

$$1 \frac{dF}{dx} = 1 \sum_j F_j (\epsilon_j F_j + \text{nonlinear terms}).$$

The net dissipation $D$ incorporates in a complicated way nonlinear transfers and “direct” dissipation/growth mechanisms represented by $\epsilon_j$.

Equations (4)–(9) are the core equations of the phase-resolving model TRIADS used in this study. The model accounts for linear processes such as refraction, shoaling, and dissipation, and for nonlinear triad interactions of shoreward propagating waves. TRIADS solves the initial-value problem 4–9 as Monte Carlo simulation consisting of a large number of individual numerical integrations (realizations). Each realization is initialized with the Fourier coefficients $B_J$ constructed at the offshore boundary by assigning uniformly-distributed random phases to modes $J$ whose amplitudes are defined by the observed directional spectrum. This approach is a straightforward numerical implementation of the random phase approximation (RPA), a fundamental assumption in the theory of wave turbulence closure, introduced at the same time as the derivation of the first kinetic wave equations (e.g., Hasselmann, 1962, 1963; Benney and Saffman, 1966; Zakharov and Filonenko, 1966, 1967;
Zakharov, 1968; Benney and Newell, 1969; also Freilich and Guza, 1984; many others; for a general discussion, see Nazarenko, 2011).

Local nonlinear transfer rates can be obtained from the model based on Equation (5) as

\[
D_J = 3 \sum_{i} W_{ij} B_{ij} \delta_{ij} P_{ij}
\]

where \(D_J\) represents the rate of energy flux transfers to mode \(J\) from the left (from lower frequencies), \(B_{ij}\) represents the rate of energy flux transfers to mode \(J\) from the right (from higher frequencies), and \(\delta_{ij}\) is the imaginary part of the complex number \(z\). The transfer rates illustrate the process of the generation of peak harmonics and IG wave, with \(D_J > 0\) in the seas band, and \(D_J > 0\) in the IG band.

4. Field observations

This study focused on the two-day period between March 3rd, 12:00 h and March 5th, 12:00 h (times reported here are UTM) which includes the most energetic storm observed during the experiment (Figs. 2 and 3).

Winds during this period were initially northward and switched direction toward the southeast on March 4th, 03:00, reaching a maximum speed of 15 m/s (Fig. 2a). At the peak of the storm on March 4th, 02:00 h, significant wave heights at sites P1 and P3 exceeded 1.5 m and 1 m, respectively (Fig. 2b). At P3, the energy at long wave band (defined as waves with periods \(<5\) s) and short wave band (defined as waves with periods \(<5\) s) followed similar patterns (Fig. 3a), with northward-propagating swell reaching a maximum significant height of 1 m on March 4th, 02:00 h (Fig. 3a).

The vertical structure of acoustic backscatter (Figs. 2d and 3b-d) and sediment concentration measurements (Fig. 2e) suggest two main phases: bed-reworking by the storm, and the Atchafalaya plume intrusion. The bed reworking includes fluidization and erosion during the storm, and deposition with fluid mud formation and consolidation after the storm. This bed-reworking phase is due to wave action: throughout March 4th, velocity measurements at P3 indicate strong bed activity, with the layer non-zero mean flow and non-zero RMS velocities estimated to extend as far as 30 cm inside the bed (Sahin et al., 2012). The intrusion of the river plume is marked by high quantities of suspended sediment on March 4th, between 12:00 h and 18:00 h. Strong south-eastward winds (from NW) during this period were adverting the Atchafalaya River sediment away from the river mouth towards the Atchafalaya Shelf. The westward currents that are dominant in the area are observed in several studies (e.g., Allison et al., 2000), to be responsible for the westward advection of this Atchafalaya River sediment plume with high sediment concentration, i.e., the resulting net sediment flux in the long-term towards the chenier-plain coast at west.

The vertical structure of SSC at P3 (Fig. 3b) suggests that the plume caused the 10 g/l SSC contour to rise to about 25 cm above the bed (Fig. 3b). The vertical structure of PC-ADP backscatter and post-storm high-concentration layer at P2 (Fig. 3c) are similar to P3. The variations of the location of acoustic backscatter maximum are relatively small at the deep site P1 (Fig. 3d). At P1, the distance between the location of maximum acoustic backscatter of the 4-MHz sensor of the ABS (dashed purple line in Fig. 3d) and the location of maximum acoustic backscatter of the 0.5-MHz sensor was used as an estimate of fluid mud thickness, which varied between 3 and 8 cm. Overall, the observations show temporal consistency among the three sites in terms of bed response to wave energy, allowing for estimating the spatial distribution of mud thickness, used in wave modeling (Section 5.1).

The waning phase of the storm, March 4th, 18:00 h to March 5th, 00:00 h, is characterized by sediment settling, due to the decreases in wave energy and wave-induced turbulence within the wave boundary layer. As a result, suspended sediment settled almost completely in only three hours during the waning phase of the storm on March 5th, between 00:00 h and 03:00 h (Figs. 2e and 3b).

The net wave dissipation rate estimated across the 8-km transect (Fig. 3e) shows an overall increasing trend throughout the storm, with two local maxima: one on March 4th, 15:00 h; and the other March 5th, 06:00 h. The latter is associated with the formation of the hindered-settling fluid-mud layer (Traykovskii et al., 2000, 2015; Sheremet et al., 2011a, Safak et al., 2013a). In contrast, the dissipation peak near March 4th, 18:00 h is intriguing. Based on the arrival time of the Atchafalaya plume (around March 4th, 12:00 h), one could argue that peak could have been associated with the increased near-bed SSC, hence with a wave-supported fluid-mud layer. However, this dissipation event appears to overlap with an earlier increase in dissipation that preceded the plume, and is probably associated with bed reworking. The spectral distribution of net dissipation (Fig. 3d) indicates that during bed-reworking, IG waves (frequency < 0.07 Hz) experienced growth, whereas the arrival of the river plume switched the regime to dissipation across all frequencies. Net wave dissipation rate during the deposition phase (4 March 12:00–5 March 06:00) was greater than the one observed during the erosion phase (4 March 00:00–12:00) which might indicate a different bed response to wave dissipation during these two phases. Significant dissipation rates are observed all the way in to the 0.3 Hz range. In spite of relatively constant peak frequency and the decrease in wave energy, wave nonlinearity (Fig. 2c) has an overall increase between 4 March 00:00–12:00 mainly due to the 1.2 m decrease in water depth (\(U_r\) is inversely proportional to the third power of depth; Section 2). At its peak, the Ursell number exceeds 0.6 at P3. This is a quite high Ursell number (Section 2) and shows the high nonlinearity which is quite remarkable for this very active dissipative wave-mud system.

5. Results

5.1. Model setup

The selection of events for numerical simulations was based on the main limitations of the numerical model: the lack of parameterizations for wind input/whitecapping for directional triads, and the lack of an adequate parameterization for dissipation induced by bed-reworking (as opposed to fluid-mud layers). The first constraint leads to eliminating time intervals with significant short wave content that could be identified as generated by local wind, e.g., having a propagation direction significantly different from swell. For this study, the directional triad-interaction wave model (Section 3) was equipped with the parameterization of dissipation due to a fluid mud layer (Ng, 2000; Kallhatu et al., 2007; Sheremet et al., 2011a). Although one can argue that the Newtonian mud model is likely inadequate for fluid mud layers, this model has the advantage that it is straightforward, quite common (e.g., Winterwerp et al., 2007; Rogers and Holland, 2009; Sheremet et al., 2011a), and provides at least an equivalent measure of the dissipation characteristics of the muddy environment. Furthermore, the thin-layer Newtonian dissipation model has been shown to be relatively skillful (e.g., Safak et al., 2013a).

Based on these considerations, two events with different wave dissipation and near-bed characteristics during the storm were selected for numerical simulations. Test A refers to observations on March 4th, 02:00, chosen as the peak of the swell energy (Fig. 3a), and minimum dissipation rates (Fig. 3e). Test B refers to observations on March 4th, 18:00 h, when we expect an overlap of bed-reworking and high near-bed SSC effects. Test B forms the central subject of this paper: compared to Test A, Test B exhibits higher dissipation rates, which makes it a better choice (stronger signal) for an in-depth analysis. Test B was also better suited for the application of this model, due to smaller incidence angles, as well as a less wind influence. In terms of currents, currents were propagating perpendicularly to the waves in Test B and there is no current effect on wave spectra (Smith, 2002). In Test A, where currents were directed onshore with waves, change in wave spectra due to currents is found to
be only about 2% (USACE, 1981). This is also supported by the findings in previous studies where co-flowing wave-current interaction was shown to incur little apparent growth of the IG band for current Froude numbers over two (Kaihatu, 2009). In addition, Kaihatu and Tahvildari (2012) further showed that mud-induced dissipation had far more influence on spectral evolution than wave-current interaction, even in high Froude number cases.

The application of Ng’s model (Ng, 2000) requires information of the thickness, viscosity and density of the fluid mud layer. For the purpose of these numerical simulations, the lutocline (upper boundary of the fluid mud layer) at P3 was defined as the location of the 10 g/L SSC contour (Fig. 3b), which yields a mud layer thickness of 3 cm during Test A and 20 cm during Test B. As discussed in Section 4, observations collected at P2 show similar patterns of vertical structure of backscatter, erosion, and accretion throughout the storm (Fig. 3c). Compared to P2 and P3, observations at P1 during the period waves were losing energy showed much smaller change in the location of the PC-ADP acoustic backscatter maximum (Fig. 3d). Based on these considerations, for Test A the mud thickness was set in the model 3-cm across the entire transect; and linearly increasing for Test B from 8-cm at P1 to 20-cm at P3.

Mud viscosity and density are related through a direct proportion based on the laboratory tests of wave forcing on mud samples from the experiment site (Robillard, 2009). Model sensitivity to mud viscosity-density pairs \((\nu_m, \rho_m)\) was tested by varying the mud viscosity within the range of previous estimates in the area \((10^{-5} - 10^{-3} \text{ m}^2/\text{s})\). Optimal values (i.e., yielding best agreement between the observed and the modeled spectra) were selected by visual inspection of frequency spectra. For Test A, the optimal values obtained were \(\nu_m = 7.4 \times 10^{-7} \text{ m}^2/\text{s}\) and \(\rho_m = 1080 \text{ kg/m}^3\).

For Test B, because observations show that mud thickness was not uniform across the shelf, spatially-varying values for \((\nu_m, \rho_m)\) were tested. The best model-data agreement was found with \(\nu_m\) linearly increasing from \(2.8 \times 10^{-5} \text{ m}^2/\text{s}\) \((\rho_m = 1060 \text{ kg/m}^3)\) at P1 to \(1.2 \times 10^{-4} \text{ m}^2/\text{s}\) \((\rho_m = 1090 \text{ kg/m}^3)\) at P3. Mud viscosity-density pairs that returned the best model-data agreement for the modeled periods (Table 2) are consistent with previous estimates at and near the study site (Sheremet et al., 2011a; Safak et al., 2013a).

Because the study area is located on the Atchafalaya clinoform (Jaramillo et al., 2009), the topography of the area is very dynamic. The large scale of the system and fast evolving bathymetry complicate attempts of numerical simulations requiring continuous updating. For this study, the bathymetry transect (Fig. 4) connecting sites P1 and P3 was reconstructed from the best data set available, that is Electronic Navigation Chart (ENC) number 11351 "Point au Fer to Marsh Island" produced by the National Oceanic and Atmospheric Administration (NOAA).

<table>
<thead>
<tr>
<th>Test</th>
<th>(h_{m3}) (cm)</th>
<th>(\nu_m) (m²/s)</th>
<th>(\rho_m) (kg/m³)</th>
<th>Spatial variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>(7.4 \times 10^{-5})</td>
<td>1080</td>
<td>Uniform</td>
</tr>
<tr>
<td>B</td>
<td>8–20</td>
<td>(2.8 \times 10^{-5} - 1.2 \times 10^{-4})</td>
<td>1060–1090</td>
<td>Linearly varying</td>
</tr>
</tbody>
</table>

**Table 2** Mud layer thickness, kinematic viscosity and density used in the numerical simulations.
Geospatial Data Abstraction Library (GDAL) code, obtained from (http://www.gdal.org), was used for extracting depth contours and point soundings from the ENC charts. The transect spans the 8 km distance between sites P1 and P3 at a bearing of 29°/C14 (clockwise from true North; Fig. 1). The tide elevation was included and a vertical offset of 0.6 m was added to the transect to account for datum change from mean lower low water to mean sea level and resultant bathymetry spans from 7.4 to 3.8 m.

The directional spectrum estimated at P1 was resampled and interpolated (see the related discussion in Davis et al., 2014) into a discrete grid of 71 frequencies by 151 alongshore wave numbers (75 up-coast wave numbers, 75 down-coast wave numbers) which give a total of 10,721 modes. The reader is referred to Section 3 for model formulation and Sheremet et al. (2016) for further details on the determination of these discretizations and related numerical performances. The results presented here represent averages over $N = 100$ realizations of shoaling transformation of the directional spectrum estimated at P1. Because the initial directional spectrum contains no information about phase correlations in the IG frequency band, IG modes were initialized with bound waves (Sheremet et al., 2016). Note that initializing TRIADS in 7.4 m of water is not ideal, because the wave field is already in shallow water regime (the linear theory yields for a wave of 8 s a wavelength of approximately 60 m), and thus it likely violates the assumption of an initial RPA state. The model provided modal amplitudes, dissipation rates, and nonlinear transfer rates at arbitrarily distributed output points (Fig. 4).

5.2. Numerical simulations

The wave spectrum is an estimate of a statistical quantity obtained with a finite number of degrees of freedom, i.e., the shape of the spectrum is only approximate. In addition, the nonlinear wave model makes its own assumptions (Section 3). Because of these reasons and because nonlinear spectral fluxes were expected to affect all frequency bands, making judgments based on the exact shapes of the simulated and observed spectral estimates is not realistic. Therefore, the search of the

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Fig. 6. Comparison of observed (left) and numerically simulated (right) cross-shore evolution of directional spectrum on March 4th, 02:00 h, Test A (a-d). a-b) P1 (initial point); c-d) P3. Comparison of observed (left) and numerically simulated (right) cross-shore evolution of directional spectrum on March 4th, 18:00 h, Test B (e-h). e-f) P1 (initial point); g-h) P3. Spectra are normalized by their peak values; color scale is logarithmic. Horizontal axis is parallel to isobaths, taken here as an approximation of the shoreline orientation (i.e., the arrow labeled “shore” is perpendicular to the shoreline). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
optimal values for the mud parameters aimed for reproducing the overall shape of the spectrum and its directional distribution, rather than the accuracy of the spectral details. Slight mismatches in the spectral peaks were considered acceptable as long as the observed overall spectral shape and variance were reproduced satisfactorily. The sensitivity of the model to mud-induced dissipation provided the necessary flexibility for this approach. Based on these criteria, model results were considered in good agreement with the observed frequency and directional spectra for Tests A and B (Figs. 5–6).

Observed directional spectra (Fig. 6) for Test A show a stronger refraction than the one computed in the numerical simulations. In Test A, the assumption of lateral uniformity in bathymetry and offshore wave conditions is likely violated due to the wide propagation angle. Because of the relatively large cross-shore extent of the simulations, wide
incidence angles involve a significant along-shore propagation distance, over which the assumption of lateral uniformity is not expected to hold.

Another possible source of uncertainty is the variability of wave conditions along the offshore boundary of the model: the model boundary conditions are based on the observations at a single point (P1), but assumed along-shore invariant.

During Test B, the dominant winds were from NW, nearly perpendicular to the shore-normal direction of swell propagation. The lack of wind parametrization in the model makes it unsuitable for describing the local wave field of short waves produced by this wind. Therefore, the performance of the model is evaluated here by comparing the simulations to the shoreward propagating components of the observed spectra. The underlying assumption that the NW wind does not affect swell propagation over the distance of integration is not unreasonable over short fetches. More importantly, it is not a fundamental assumption of the model, but just a convenient one, until a wind-input parametrization becomes available.

Both the directional and the frequency spectra show weak effects that could be attributed to nonlinear interactions, such as marginal IG- and seas-band growth (Figs. 5–6) but the evidence is not convincing. Concluding, however, that wave transformation is essentially linear, based on the strength of the spectral density evolution alone, ignores the complexity of the interplay between nonlinearity and dissipation. Both Test A and Test B correspond to periods of time characterized by $U_r = 0.45$ at P3 (Fig. 2c). This suggests that wave nonlinearity plays a role, possibly obscured by dissipation. A closer look into the mechanics of the model helps clarify this question.

The evolution of cross-shore energy fluxes (Fig. 7) is dominated by dissipation, with the total, swell and seas energy flux decaying by at least a factor of 2 between P1 and P3 for Test A and a factor of 5 for Test B (Fig. 7c,i). As expected, Test B is significantly more dissipative (see also Fig. 8). The model captures well the trends for the total, swell and seas evolution. As seen from the net dissipation rates (Fig. 3f) the IG band shows a slight growth in Test A and dissipates in Test B. The model qualitatively captures the trend but its predictions for the IG band are biased high, possibly due to an initial overestimation of the initial IG bound-wave variance, or an inadequate description of wave-mud interaction at IG frequencies.

The model includes a depth-limited breaking mechanism, but for Tests A and B dissipation rates associated with wave breaking were zero to the numerical precision of the integration (not shown). For both tests, mud-induced dissipation rates are overall one order of magnitude larger than nonlinear spectral transfers. Despite this, nonlinearity is active, with a recurrence, “chatter”-like aspect (nonlinear oscillations, Figs. 9 and 10). The difference between the nonlinear trends between Tests A and B is subtle, but consistent.

In Test A (weaker dissipation, Fig. 9) the nonlinear “chatter” is more intense and fits better the expected trend of overall transfers from the peak (swell) to higher frequencies (seas). In the first half of the domain (first 4 km), the swell and IG bands are active, with the swell band transferring energy to the IG band: the blue line in panel f is in the negative domain (loss), matching the positive red line in panel g (gain). The seas are largely inactive, with no exchange with the swell: the pink line in panel f hovers near zero, as does the blue line in panel g. In the second half of the domain (last 4 km), seas activate and transfers begin to balance: under heavy mud-induced dissipation swell begins to drain energy from the seas band, and decreases transfers to the IG band. Toward the shallow end of the transect nonlinear transfers decay

![Figure 9](image-url)
significantly.

Test B (stronger dissipation, Fig. 10) shows a different picture. As expected, dissipation suppresses the nonlinear chatter, but not completely. In the first half of the transect swell still transfers energy to the IG band as before. The biggest difference is in the behavior of the seas band as it becomes active. Statistically, nonlinear spectral fluxes are directed from low to high frequencies. In our simulations, however, instead of extracting energy from swell, or at least staying at a zero-transfer balance, the seas band consistently transfers energy to the swell throughout the entire domain. The net energy transfers appear to be consistently directed toward the lower frequencies.

For a qualitative explanation of spectral-flux reversal, consider the analytical solution of a single difference triad (with frequencies $df$, $f$, $f + df$, where $df \ll f$) evolving without dissipation over a flat bottom. Craik (1985) shows that there is a cyclic transfer of energy between the three components, with the nonlinear wavelength $k(f + df) - k(f) - k(df)$. Assuming that a small dissipation rate is added, such as to affect only the long wave, because the three modes communicate the eventual effect of dissipation will be to drain the entire energy in the system. If the dissipation is small enough, the evolution is nearly periodic, similar to the nondissipative system. However, at every exchange cycle a small quantity is lost by the long wave. On average, this establishes a net energy flux toward the dissipative mode. A more in-depth analysis based on numerical simulations can be found in Sherm et al. (2005b).

Although flux reversal is obvious in the simple case of a single wave triad, in field observations the effect is obscured and weakened by frequency-distributed dissipation, and the participation in the process of the entire wave spectrum. A simple test of this interpretation can be done by completely removing the mud-induced dissipation in the numerical models.
simulations. The results of this “alternative” Test B are shown in Figs. 11 and 12. In the absence of mud-induced dissipation, breaking becomes more active at the end of the transect (Fig. 11). The alternative Test B shows an evolution (Fig. 12) similar to Test A: throughout the domain the swell band pumps energy into the IG band; at the shallow end of the transect, the swell begins to transfer energy to seas, as wave-breaking switches on. Based on these comparisons, we interpret the behavior of Test B (with mud) as evidence that, in the presence of mud-induced dissipation, nonlinear interactions transfer energy from short waves into the longer wave bands, where it is dissipated by direct interaction with the muddy bed. Therefore, mud induced dissipation effectively drains the entire spectrum, and not just the bands in which it acts directly on. This is consistent with the hypothesis first proposed by Sheremet and Stone (2003), and later demonstrated by Sheremet et al. (2005b), Kaihatu et al. (2007) and Elgar and Raubenheimer (2008), that the presence of mud-induced dissipation that acts within a certain frequency band can reverse energy transfers. Note that, despite the positive transfer rates, the spectral peak shows a net decay.

As suggested by Kaihatu et al. (2007), the spectral-flux reversal and subsequent decay of the high-frequency spectral band imply a reduction in the nonlinearity of the wave field. Indeed, energy damping and loss of phase correlation with the spectral peak indicate the waning of higher-order statistics (e.g., skewness and asymmetry), and an overall approach to a Gaussian, linear wave field. Remarkably, the decrease in the nonlinearity of the wave field is brought about precisely by the intrinsic nonlinear character of the system, which allows the exchange of energy between the high frequency bands (not directly affected by mud-induced dissipation) and lower frequencies, where mud-induced dissipation acts as a sink.

6. Summary and discussion

Based on high-resolution field observations collected in recent years, a number of studies have shed some light on the bed-reworking process by waves in muddy environments. This study takes advantage of this progress, as well as the newly developed phase-resolving numerical model for nonlinear shoaling of directional spectra, to revisit the problem of the interplay between nonlinear interactions and band-limited mud-induced dissipation.

High-resolution field observations of flows and sediment transport collected in 2008 on the muddy Atchafalaya inner shelf captured the details of bed reworking by waves. They show the initially stable seafloor undergoing a sequence of transformations that include expansion, liquefaction, fluidization, and resuspension, followed by the intrusion of the sediment-laden Atchafalaya plume and, in the waning stage of the storm, rapid settling and fluid mud formation (Sahin et al., 2012). The spatial and frequency distribution of net wave dissipation showed a consistently increasing trend, with significant dissipation during the bed-reworking process, but reaching its maximum rates coinciding with the formation of fluid mud layers. Using the simple Newtonian
parameterization proposed by Ng (2000), the nonlinear phase-resolving model developed by Davis et al. (2014) and Sheremet et al. (2016) was able to simulate the spatial transformation of the directional spectra during two test time intervals of the storm: coinciding, one with peak swell energy, and the other with the thickest mud layer estimated (~20 cm). Mud parameters inverted from the model tests are consistent with former observations and estimates near the study site.

Numerical simulations presented here identify the dissipation-induced shift in nonlinear spectral transfers, hypothesized by Sheremet and Stone (2003) and supported by the findings of Sheremet et al. (2005b), Kaihatu et al. (2007) and Elgar and Raubenheimer (2008), that causes wave energy to drain from the entire spectrum, and not just the frequency bands (long waves) that interact directly with the bed. A remarkable consequence of the coupling between the mechanisms of mud-induced dissipation and nonlinear exchange is the damping of high-frequency waves, otherwise not directly affected by the presence of mud. The damping of the high-frequency waves reduces wave nonlinearity, as measured by higher-order statistics (skewness, asymmetry, etc.) and restores the Gaussian, linear character of the wave field. This study also highlights significant gaps in our understanding of the wave-mud interaction process (e.g., the lack of a formulation for the detailed dynamics of bed-rewetting: liquefaction, fluidization, etc.), observation shortcomings (most notably spatial density of data collected; the lack of deep water observations), and modeling limitations (e.g., the lateral uniformity assumption; the lack of adequate parameterizations for wind input/whitecapping, etc.). Numerical simulations performed here serve mostly to understand the inner mechanisms of wave-mud interaction. Further significant field and theoretical efforts are needed to advance our understanding of its physical details.

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References